PHYS 483 Problem Set 3

Date: Tuesday, March 18th, 2003
Due date: Tuesday, March 25th, 2003

- **Problem 1** [15 points] *Introduction to Algorithms, page 31, exercise 2.1-2*
Show that for any real constants a and b, where $b > 0$,
$(n + a)^b = \Theta(n^b)$
- **Problem 2** [15 points] *Introduction to Algorithms, page 31, exercise 2.1-4*
Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?
- **Problem 3** [20 points] *Introduction to Algorithms, page 58, exercise 4.1-6*
Solve the recurrence $T(n) = 2T(\sqrt{n}) + 1$ by making a change of variables. Do not worry about whether values are integral.
- **Problem 4** [25 points] *Introduction to Algorithms, page 61, exercise 4.2-5*
Use a recursion tree to solve the recurrence $T(n) = T(\alpha n) + T((1 - \alpha)n) + n$, where α is a constant in the range $0 < \alpha < 1$.
- **Problem 5** [25 points] *Introduction to Algorithms, page 64, exercise 4.3-2*
The running time of an algorithm A is described by the recurrence $T(n) = 7T(n/2) + n^2$. A competing algorithm A' has a running time of $T'(n) = aT'(n/4) + n^2$. What is the largest integer value for a such that A' is asymptotically faster than A?