PHYS 483 Problem Set 5

Date: Tuesday, April1st, 2003
Due date: Tuesday, April 8th, 2003

• **Problem 1** *Introduction to Algorithms, page 170, problem 8-5*
Median-of-3 partition

One way to improve the RANDOMIZED-QUICKSORT procedure is to partition around an element x that is chosen more carefully than by picking a random element from the subarray. One common approach is the **median-of-3** method: choose x as the median (middle element) of a set of 3 elements randomly selected from the subarray. For this problem, let us assume that the elements in the input array $A[1 \ldots n]$ are distinct and that $n \geq 3$. We denote the sorted output array by $A'[1 \ldots n]$. Using the median-of-3 method to choose the pivot element x, define $p_i = \Pr\{x = A'[i]\}$.

(a) [35 points] Give an exact formula for p_i as a function of n and i for $i = 2, 3, \ldots, n-1$. (Note that $p_1 = p_n = 0$.)

(b) [20 points] By what amount have we increased the likelihood of choosing $x = A'[\lfloor (n+1)/2 \rfloor]$, the median of $A[1 \ldots n]$, compared to the ordinary implementation? Assume that $n \to \infty$, and give the limiting ratio of these probabilities.

(c) [40 points]If we define a "good" split to mean choosing $x = A'[i]$, where $n/3 \leq i \leq 2n/3$, by what amount have we increased the likelihood of getting a good split compared to the ordinary implementation? (*Hint:* Approximate the sum by an integral.) (*Instructor's note: It is implicit that this question, too, should be answered for the case where* $n \to \infty$.)

(d) [5 points] Argue that the median-of-3 method affects only the constant factor in the $\Omega(n \lg n)$ running time of quicksort.