CSE 505 Problem Set 3

Date: Thursday, October 16th, 2003
Due date: Thursday, October 23rd, 2003

- **Problem 1**

An ambulance travels back and forth, at constant specific speed v, along a road of length l. In other words, at any moment in time, consider the location of the ambulance to be uniformly distributed over the interval $(0, l)$. Also at any moment in time, an accident (not involving the ambulance itself) occurs at a point uniformly distributed on the road; that is, the accident's distance from one of the fixed ends of the road is also uniformly distributed over the interval $(0, l)$. Assume the location of the accident and the location of the ambulance are independent.

a. Supposing the ambulance is capable of *immediate* U-turns, compute the CDF and PDF of the ambulance's travel time T to the location of the accident.

b. Repeat part **a**, but now suppose U-turns are only possible at either fixed end of the road.

- **Problem 2**

Random variable X is described by the PDF

$$f_X(x) = \begin{cases} 0.1 & ,0 \leq x \leq 10.0 \\ 0 & ,\text{otherwise} \end{cases}$$

Another random variable Y, is defined by $Y = -\ln X$. Determine the PDF $f_Y(y)$.

- **Problem 3**

Let $X_1, X_2, \ldots, X_n$ where $n \geq 2$, be independent and identically distributed continuous random variables with CDF $F(x)$ and PDF $f(x)$. Define $Y = max(X_1, \ldots, X_n)$, $Z = min(X_1, \ldots, X_n)$ and $D = Y - Z$.

a. Show that:

$$F_{Y,Z}(y, z) = P(Y \leq y, Z \leq z) = \begin{cases} F(y)^n - [F(y) - F(z)]^n & ,y > z \\ F(y)^n & ,y \leq z \end{cases}$$

b. Show that:

$$F_D(d) = P(D \leq d) = \begin{cases} n \int_{-\infty}^{\infty} [F(y) - F(y-d)]^{n-1} dy & ,d \geq 0 \\ 0 & ,d < 0 \end{cases}$$