

# CSE 505 Problem Set 3

*Date: Thursday, October 9<sup>th</sup>, 2003*  
*Due date: Thursday, October 16<sup>th</sup>, 2003*

## • Problem 1

Let  $x_1$ ,  $x_2$ , and  $x_3$  be three independent experimental values of a particular continuous random variable  $X$ . Given that  $x_1$  is greater than  $x_2$ , what is the conditional probability that  $x_1$  is also greater than  $x_3$ ?

## • Problem 2 : Gaussian Integrals

a. The “basic” Gaussian integral is:

$$\int_{-\infty}^{+\infty} e^{-x^2} dx$$

Show that the value of this integral is  $\sqrt{\pi}$ . This can be done as follows. Let  $I = \int_{-\infty}^{+\infty} e^{-x^2} dx$ . Then,

$$I^2 = \int_{-\infty}^{+\infty} e^{-x^2} dx \cdot \int_{-\infty}^{+\infty} e^{-y^2} dy$$

and

$$I^2 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-(x^2+y^2)} dx dy$$

Consider this to be an integral over all  $x - y$  plane. Now, change to polar coordinates, and do a suitable change of variables to reach the required result.

b. Show that

$$\int_{-\infty}^{+\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

c. Show that

$$\int_{-\infty}^{+\infty} e^{-a(x-b)^2} dx = \sqrt{\frac{\pi}{a}}$$

d. Finally, show that

$$\int_{-\infty}^{+\infty} e^{-(ax^2+bx+c)} dx = \sqrt{\frac{\pi}{a}} \cdot e^{\frac{b^2-4ac}{4a}}$$

*Hint: Complete the square in the exponent.*

• **Problem 3**

A continuous random variable  $x$  is said to be a *Gaussian random variable* if it is distributed as:

$$f_x(x_0) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \cdot e^{-\frac{(x_0 - \bar{x})^2}{2\sigma_x^2}}$$

It can be shown that the expectation and variance of the random variable  $x$  in this case are  $\bar{x}$  and  $\sigma_x^2$ , respectively. It can also be shown that this PDF is properly normalized.

For this problem, assume that  $x$  is a zero-mean ( $\bar{x} = 0$ ) Gaussian random variable with variance  $\sigma^2$  ( $\sigma_x^2 = \sigma^2$ ). Derive an expression for  $E[x^n]$ , the  $n$ th moment, that is valid for all integer  $n \geq 0$ .

*Hint: Consider the two cases of  $n$  odd, and  $n$  even.*

• **Problem 4**

To do this problem, you might find the results of problem 3 useful.

A *central Chi-Square* random variable  $y$ , with  $n$  degrees of freedom, is defined as:

$$y = x_1^2 + x_2^2 + \cdots + x_n^2$$

where the  $x_i$ 's are independent and identically distributed zero-mean Gaussian random variables with variance  $\sigma^2$ . Find the expectation and variance of  $y$ .