

CSE 505 Final Examination

Date: Tuesday, January 6th, 2004

Duration: Two hours

• Problem 1

Joe is performing an experiment by repeatedly rolling a pair of six-sided dice.

- a. What is the probability that he rolls a total of seven before he rolls a total of eight?
- b. He calls a roll a success if the numbers on the two dice are equal. Find the PMF for L , the number of rolls up to and including the sixth success.
- c. Calculate the expectation value for M , the number of successes in 1000 rolls.

• Problem 2

On a computer network, there are three computers, A, B, and C, that can generate data packets. Each computer produces data packets in a Poisson manner, with rates λ_A , λ_B , and λ_C , respectively.

- a. Assume we listen to packets on the network, disregarding the sources of packets. Calculate the PDF for L , the interarrival times of any two packets.
- b. If we start listening to packets on the network at a random time, what is the probability that the first packet we catch is from computer A?
- c. A packet from computer B is an ICMP packet with probability P . (Only computer B produces ICMP packets.) What is the PDF for interarrival times of ICMP packets on the network?

• Problem 3

a. A discrete-state discrete-transition Markov process has three states. The state transition probabilities at each step from state i to state j are given by the state transition matrix p_{ij} . It is known that $p_{ij} = p_{ji} > 0$ for all pairs (i, j) . Find the limiting state probabilities P_i in terms of p_{ij} .

b. This time, consider a Markov process with N states, with the same property that $p_{ij} = p_{ji} > 0$. Find the limiting state probabilities P_i in terms of p_{ij} . (The result is very important for quantum statistical mechanics.)

• Problem 4

Two light bulbs, A and B, illuminate a room with no windows or other light source. Bulb A has an exponentially distributed lifetime, with an expectation of 1000 hours. In case bulb A fails, the attendant replaces the bulb. The time it takes to replace the bulb is (believe it or not!) also exponentially distributed, with an expected value of 50 hours. Bulb B is a special, long-lasting bulb that also has an exponentially distributed lifetime with an expected value of 5000 hours. However, if bulb B fails, a specialist must be called to replace it, and the replacement time for bulb B is exponentially distributed with an expected value of 500 hours.

- a. Draw a Markov model for this process. Mark your states and transitions clearly.
- b. What fraction of the time is the room in total darkness?
- c. What fraction of the time is the room illuminated with two bulbs?
- d. Which of the two bulbs illuminates the room on its own for longer stretches of time?