

# CSE 505 Final Examination

*Date: Tuesday, January 6<sup>th</sup>, 2004*

*Duration: Two hours*

## • Problem 1

Joe is performing an experiment by repeatedly rolling a pair of six-sided dice.

- What is the probability that he rolls a total of seven before he rolls a total of eight?
- He calls a roll a success if the numbers on the two dice are equal. Find the PMF for  $L$ , the number of rolls up to and including the sixth success.
- Calculate the expectation value for  $M$ , the number of successes in 1000 rolls.

## • Problem 2

On a computer network, there are three computers, A, B, and C, that can generate data packets. Each computer produces data packets in a Poisson manner, with rates  $\lambda_A$ ,  $\lambda_B$ , and  $\lambda_C$ , respectively.

- Assume we listen to packets on the network, disregarding the sources of packets. Calculate the PDF for  $L$ , the interarrival times of any two packets.
- If we start listening to packets on the network at a random time, what is the probability that the first packet we catch is from computer A?
- A packet from computer B is an ICMP packet with probability  $P$ . (Only computer B produces ICMP packets.) What is the PDF for interarrival times of ICMP packets on the network?

## • Problem 3

a. A discrete-state discrete-transition Markov process has three states. The state transition probabilities at each step from state  $i$  to state  $j$  are given by the state transition matrix  $p_{ij}$ . It is known that  $p_{ij} = p_{ji} > 0$  for all pairs  $(i, j)$ . Find the limiting state probabilities  $P_i$  in terms of  $p_{ij}$ .

b. This time, consider a Markov process with  $N$  states, with the same property that  $p_{ij} = p_{ji} > 0$ . Find the limiting state probabilities  $P_i$  in terms of  $p_{ij}$ . (The result is very important for quantum statistical mechanics.)

## • Problem 4

Two light bulbs, A and B, illuminate a room with no windows or other light source. Bulb A has an exponentially distributed lifetime, with an expectation of 1000 hours. In case bulb A fails, the attendant replaces the bulb. The time it takes to replace the bulb is (believe it or not!) also exponentially distributed, with an expected value of 50 hours. Bulb B is a special, long-lasting bulb that also has an exponentially distributed lifetime with an expected value of 5000 hours. However, if bulb B fails, a specialist must be called to replace it, and the replacement time for bulb B is exponentially distributed with an expected value of 500 hours.

- Draw a Markov model for this process. Mark your states and transitions clearly.
- What fraction of the time is the room in total darkness?
- What fraction of the time is the room illuminated with two bulbs?
- Which of the two bulbs illuminates the room on its own for longer stretches of time?